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S-Duality and noncommutative gauge theory

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Abstract: It is conjectured that strongly coupled, spatially noncommutative $\mathcal{N} = 4$ Yang-Mills theory has a dual description as a weakly coupled open string theory in a near critical electric field, and that this dual theory is fully decoupled from closed strings. Evidence for this conjecture is given by the absence of physical closed string poles in the non-planar one-loop open string diagram. The open string theory can be viewed as living in a geometry in which space and time coordinates do not commute.

Keywords: D-branes, String Duality, Non-Commutative Geometry.

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1. Introduction

Noncommutative field theories have a rich and fascinating structure. The embedding of these theories into string theory [1] suggests that this structure may be directly relevant to understanding the inevitable breakdown of our familiar notions of space and time at short distances in quantum gravity.

Investigations to date have largely concentrated on theories with purely spatial noncommutativity (see however [2]). While such theories are interestingly nonlocal in space, they are local in time, admitting familiar notions like that of the hamiltonian and a quantum state. Noncommutativity of a time-like coordinate should have even more far-reaching consequences, and it is natural to ask whether or not such theories exist.

In this paper we give one answer to this question by asking another: What is the strong coupling dual of NCYM (spatially-noncommutative $\mathcal{N} = 4$ Yang-Mills)? This question can be addressed in the description of NCYM as a scaling limit of three-branes with a B field in IIB string theory [3]. IIB S-duality induces an S-duality on the NCYM theory, mapping the strongly coupled NCYM theory to a weakly coupled open string theory.[1] This open string theory can be viewed either as living in a near critical electric field,[2,3] or in a space-time with noncommuting space and time coordinates. A precise statement of the spacetime noncommutativity in this theory is that the temporal zero mode X^0 on the open string worldsheet does not commute with the spatial zero modes. The scale associated with this noncommutativity is the same as the effective open string scale. Thus the effects of the noncommutativity are inextricably tied up with the usual stringy nonlocalities.

Since the closed string sector of the IIB theory is decoupled in the scaling limit, the dual open string theory does not have a closed string sector. The appearance of an open string theory without a closed string sector is striking. Ordinarily closed string poles appear in open string loop diagrams, and unitarity then requires the addition of asymptotic closed string states. In order to better understand this point we analyze (following [11]–[16]) the nonplanar one loop open string diagram for the bosonic case. We find that the temporally noncommutative phases lead to a precise cancellation of all the closed string poles, in accord with our expectations. This cancellation in fact occurs for branes of any dimension, indicating the existence of a family of non-commutative open string theories.

This paper is organized as follows. In section 2 we derive the S-dual of NCYM, which we refer to as NCOS (noncommutative open strings), by embedding in string theory. In section 3 we show that it is a decoupled open string theory with a near-critical electric field. In section 4 we give evidence at the one loop level for the decoupling of closed strings by computing the non-planar annulus for bosonic string theory with two incoming and two outgoing tachyons. Section 5 contains a preliminary analysis of the general higher loop diagram; no obvious closed string singularities are found. In section 6 we make some comments regarding the supergravity duals of our open string theory. We conclude with some discussion in section 7. For simplicity we concentrate on the U(1) theories but our results generalize easily to U(N).

Related work will appear in [17].

[1]The low energy sector of the open string theory is ordinary $\mathcal{N} = 4$ YM, and the induced duality reduces to the standard S-duality.

[2]The existence of a scaling theory at near critical electric fields, and its relevance to temporal noncommutativity was emphasized to us by N. Seiberg, L. Susskind and N. Toumbas (private communications). The scaling to the critical electric field was also considered in [4, 5].

[3]The critical value of the electric field arises when the force pulling apart the charges at either end of the string just balances the string tension, so that the string is effectively tensionless [6]–[10]. Beyond this value the spectrum contains a tachyon and the vacuum is unstable.

2. Inducing S-duality

The Olive-Montonen dual of ordinary $\mathcal{N}=4$ SYM may be deduced as a consequence of the S duality of IIB theory in the presence of D3-branes in the zero slope limit. In this section we will determine the Olive-Montonen dual of spatially noncommutative $\mathcal{N}=4$ SYM, using the S duality of IIB theory in flat space in the presence of D3-branes and a background $B_{\mu\nu}$ field, together with the modified zero slope limit [3].

Consider a D3-brane, extended in the $0,1,2,3$ directions, in a background geometry

$$g'_{\mu\nu}=\eta_{\mu\nu}\,,\qquad g'_{ij}={\alpha'}^2k_1\delta_{ij}\,,\qquad g'_{MN}=\delta_{MN}\,,\qquad B'_{ij}=-B\epsilon_{ij}\,,\qquad g'_{\rm str}=\alpha' k_2\,. \tag{2.1}$$

in the limit $\alpha'\to 0$, keeping k_1,k_2,B fixed (we will refer to this as the NCYM limit). Here $\mu,\nu=0,1$ with $i,j=2,3$ and $M,N=4,\ldots,9$. (We will reserve unprimed notation for the S-dual variables to be introduced in the next sub-section.) It was shown in [3] that the decoupled theory on the brane is noncommutative U(1) SYM propagating on a four dimensional space with (open string) metric (we use the conventions of [3]) $G'_{\mu\nu}=\eta_{\mu\nu}$, $G'_{ij}=\frac{(2\pi B)^2}{k_1}\delta_{ij}$, noncommutativity parameter $\theta'^{ij}=\epsilon^{ij}/B$, and gauge coupling $g^2_{\rm YM}=2\pi G'^2_o$, where $G'^2_o=2\pi k_2B/k_1$. In order to obtain noncommutative field theory propagating on a space with unit metric we choose $k_1=(2\pi B)^2$. In terms of the field theory couplings θ' and G'_o, $B=1/\theta'$ and $k_2=(2\pi)G'^2_o/\theta'$.

In order to obtain a weakly-coupled dual description of the noncommutative gauge theory at large G'_o we will consider the NCYM limit described above in an S-dual picture. Before describing this in detail we note that the S-dual version has two potentially unpleasant features:

a) It seems to involve branes in the presence of an an RR 2 form potential (the S-dual of B'_{ij}).

b) The S-dual of the NCYM limit takes the closed string coupling $g_{\rm str}$ to infinity, seeming to indicate that any description of brane dynamics obtained in this picture will be strongly rather than weakly coupled, independent of G_o.

These difficulties may both be circumvented. In order to avoid having to deal with RR fields, we gauge away the constant bulk NS-NS potential before performing the S-duality. This gauge transformation induces a magnetic field $F'_{23}=B$ on the the D3-branes, which is converted into an electric field by the S-duality; in fact an electric field that approaches its critical value in the scaling limit. This electric field may in turn be gauged into a constant background NS-NS two form potential $B_{01}=F_{01}$ in the bulk. But, in such a background, the open string coupling that governs the strength of interactions between brane modes is not directly related to the closed string coupling. It turns out that the open string coupling in this background is $G_o=1/G'_o$, i.e. it is the inverse of the original open string coupling, and therefore remains finite despite the fact that $g_{\rm str} \to \infty$. Thus at large G'_o, the effective description is a weakly coupled noncommutative open string theory, with noncommutativity in the time direction!

We now consider this limit in more detail. We could consider any finite number of branes, N, but we will mostly stick to the case $N = 1$ for simplicity.

2.1 Born-Infeld S-duality

S-duality transforms a constant magnetic field on the three-brane to a constant electric field. Constant fields on a single D3-brane are governed by the Born-Infeld action

$$S_{BI} = \frac{1}{(2\pi)^3 \alpha'^2 g_{\rm str}} \int d^4x \sqrt{-\det(g_{\mu\nu} - 2\pi\alpha' F_{\mu\nu})}\,. \tag{2.2}$$

The action of S duality on S_{BI} will be reviewed in this subsection (see [18]). Consider a gauge theory on a torus. The flux of the magnetic field on any nontrivial two cycle of the torus is integrally quantized, and so must, under electromagnetic S-duality, map to a quantized electric flux. Recall why electric flux on a torus is quantized. The constant piece (zero momentum mode) of a gauge field in flat infinite space is physically unmeasurable, as it can be gauged away. This is not true, however, on a torus, as the Wilson line $e^{i\int A\,dx}$ over any nontrivial cycle of the torus is a gauge invariant observable, implying that the zero momentum piece of the gauge field A_i is a periodic physical 'coordinate', with period $2\pi/L_i$ (L_i is the size of the i^{th} spatial direction). Consequently, the momentum conjugate to the zero mode of A_i is quantized in integral units of L_i. This quantized momentum is the electric flux that is interchanged with the quantized magnetic flux under S duality.

In order to work out the expression for the quantized electric flux, consider the theory (2.2) on a rectangular torus, with spatial coordinate radii L_1, L_2, L_3. We are interested in background field configurations in which F_{01} is nonzero and constant, but F_{ij} is zero. Since $\dot{A}_1$ appears in the lagrangian only through F_{01}, it is sufficient, for the purposes of computing canonical momenta in such backgrounds, to set F_{ij} to zero in the lagrangian. For a diagonal metric the Born Infeld action simplifies to (recall g^{00} is negative)

$$S = \frac{1}{(2\pi)^3 \alpha'^2 g_{\rm str}} \int d^4x \sqrt{-g}\,\sqrt{1 + (2\pi\alpha')^2 g^{11} g^{00} F_{01}^2}\,. \tag{2.3}$$

Thus, for constant F_{01}, the momentum conjugate to A_1 is

$$P^1 = N L_1 = \frac{1}{2\pi g_{\rm str}} L_1 L_2 L_3 \sqrt{-g}\, \frac{g^{11} g^{00} F_{01}}{\sqrt{1 + (2\pi\alpha')^2 g^{11} g^{00} F_{01}^2}}\,. \tag{2.4}$$

Thus the constant F'_{23} background of the spatially noncommutative theory maps, under S duality, to a background with constant F_{01}, whose value is given by the solutions to the equations

$$\frac{\sqrt{-g}}{g_{\rm str}} \frac{g^{11}g^{00}F_{01}}{\sqrt{1+(2\pi\alpha')^2 g^{11}g^{00}F_{01}^2}} = F'_{23} = \frac{1}{\theta'}\,, \tag{2.5}$$

where $g_{\mu\nu}$ and $F_{\mu\nu}$ are the background metric and field strength in the S dual description. In terms of the critical value of the electric field

$$F_{01}^{\rm crit} = \frac{\sqrt{-g_{00}g_{11}}}{2\pi\alpha'} \tag{2.6}$$

one finds

$$F_{01} = \frac{F_{01}^{\rm crit}}{\sqrt{1+g_{22}g_{33}(\theta'/2\pi\alpha' g_{\rm str})^2}}\,. \tag{2.7}$$

2.2 The scaling limits

Consider IIB theory with a D3-brane in the presence of a background NS-NS 2-form potential, $B_{\mu\nu}$. Prior to any scaling limit, an open string metric $\widetilde{G}^{AB}$ (the symbol G^{AB} will be reserved for a rescaled open string metric defined below) and a non-commutativity parameter Θ can be deduced from disk correlators on the open string worldsheet boundaries

$$X^A(0)X^B(\tau) = -\alpha'\widetilde{G}^{AB}\ln(\tau)^2 + \frac{i}{2}\Theta^{AB}\epsilon(\tau)\,, \qquad A,B=0,1,2,3\,. \tag{2.8}$$

The open string coupling G_o is similarly read off from the coefficient of the gauge theory action. These are related to closed string quantities by the formulae [3]

$$\begin{aligned} 2\pi\alpha'\widetilde{G}^{AB} + \Theta^{AB} &= (2\pi\alpha')\left(\frac{1}{g+2\pi\alpha' B}\right)^{AB}\,, \\ G_o^2 &= g_{\rm str}\frac{\det^{1/2}(g+2\pi\alpha' B)}{\det^{1/2}(g)}\,. \end{aligned} \tag{2.9}$$

As discussed above, in the NCYM limit, $\alpha'\to 0$ while the open string metric G'^{AB}, open string coupling G'_o and the (spatial) non-commutativity matrix Θ'^{AB} are held fixed. We would now like to study this scaling limit in the S-dual description of type-IIB theory. We will call this the NCOS limit. Under an S-Duality, the type-IIB closed string backgrounds transform in the usual fashion, $g'_{\rm str} = 1/g_{\rm str}$, $g'_{\mu\nu} = g_{\mu\nu}/g_{\rm str}$ (α' is unchanged). The associated open string quantities may then be read from their definitions in (2.9). The results, in the limit $\alpha'\to 0$, are summarized in table 1.

Here

$$\mu,\nu=0,1\,,\qquad i,j=2,3\,,\qquad A,B=0,1,2,3\,,\qquad M,N=4,5,6,7,8,9\,.$$

In table 1 we have expressed all open and closed string quantities as functions of θ and G_o, the noncommutativity parameter and open string coupling in the (S-dual) NCOS theory. We have also defined the quantities, $\alpha'_{\rm eff}$ the effective open string scale and the rescaled open string metric $G^{AB} = \frac{\alpha'}{\alpha'_{\rm eff}}\widetilde{G}^{AB}$ of the NCOS theory.

The NCYM limit	The S-dual NCOS limit
$g^{'}_{\mu\nu} = \eta_{\mu\nu}$	$g_{\mu\nu} = \frac{\theta G_o^4}{2\pi\alpha'}\eta_{\mu\nu}$
$g^{'}_{ij} = \frac{(2\pi\alpha')^2}{\theta'^2}\delta_{ij}$	$g_{ij} = \frac{2\pi\alpha'}{\theta}\delta_{ij}$
$B'_{\mu\nu} = F'_{\mu\nu} = 0$	$B_{\mu\nu} = F_{\mu\nu} = F^{\rm crit}_{\mu\nu}\left(1 - \frac{1}{2}\left(\frac{2\pi\alpha'}{\theta G_o^2}\right)^2\right)$
$B'_{ij} = F'_{ij} = -\frac{1}{\theta'}\epsilon_{ij}$	$B_{ij} = F_{ij} = 0$
$g^{'}_{\rm str} = G_o^{'2}\frac{2\pi\alpha'}{\theta'}$	$g_{\rm str} = \frac{\theta'}{G_o^{'2}2\pi\alpha'} = \frac{G_o^4\theta}{2\pi\alpha'}$
$G'^{AB} = \eta^{AB}$	$\frac{\alpha'}{\alpha'_{\rm eff}}\widetilde{G}^{AB} \equiv G^{AB} = \eta^{AB}$
$G'^{MN} = g'^{MN} = \delta^{MN}$	$G^{MN} = g^{MN} = \frac{2\pi\alpha'}{\theta G_o^4}\delta^{MN}$
$\Theta'^{\mu\nu} = 0$	$\Theta^{\mu\nu} = -\theta' G_o^{'2}\epsilon^{\mu\nu} = -\theta\epsilon^{\mu\nu}$
$\Theta'^{ij} = -\theta'\epsilon^{ij}$	$\Theta^{ij} = 0$
$G^{'}_o = G^{'}_o$	$G_o = \frac{1}{G'_o}$
$\alpha' = \alpha'$	$\alpha'_{\rm eff} = \frac{\theta}{2\pi}$

Table 1

Note that

1. In the limit $\alpha'\to 0$, the electric field F_{01} of the NCOS theory attains its critical value
$$F_{01}^{\rm crit} = \frac{\theta G_o^4}{(2\pi\alpha')^2}\,. \tag{2.10}$$

2. The energy per unit coordinate length of an NCOS open string stretched in the 1 direction is given by (recall that the ends of an open string are charged)
$$p_0 = \frac{\epsilon_{01}}{2\pi}\left(\frac{1}{\alpha'} - 2\pi\epsilon^{01}F_{01}\right)\Delta x^1 = \frac{1}{4\pi\alpha'_{\rm eff}}\Delta x^1 \tag{2.11}$$
so these open strings have an effective tension set by $\alpha'_{\rm eff}$. As a consequence, it will turn out that in the NCOS limit excited open string oscillator states are part of the decoupled theory on the brane in the NCOS limit, and that their mass scale is also set by $\alpha'_{\rm eff}$.

3. The open string coupling G_o is the inverse of the gauge coupling G'_o in the NCYM limit.

To summarize, strongly coupled spatially noncommutative Yang-Mills theory has an effective description as a weakly coupled open string theory living on D3-branes, in the presence of a near critical electric field. The parameters of this open string theory are listed in table 1. We will explore this theory in the rest of this paper.

3. The classical NCOS theory

3.1 Spacetime noncommutativity

In the NCOS limit, open strings on the brane propagate in a background electric field. This results in temporal noncommutativity, in the sense that the open string zero modes obey

$$[X^\mu, X^\nu] = i\theta\epsilon^{\mu\nu}\,, \tag{3.1}$$

as may easily be seen from (2.8).

Disk diagrams in the NCOS theory are very simple. As argued in [19, 3], open string correlation functions on the disk in the NCOS theory may be obtained from the equivalent correlation functions in the theory without the electric field, by the addition of noncommutative phases in the $0,1$ directions (and using the appropriate open string metric and coupling). Thus the classical action for open string modes in the NCOS limit may be obtained by turning all products in the usual open string classical action into star products. In other words if we think about the open string field theory action $S = \int AQA + A *_w A *_w A$ there the $*_w$ product is Witten's star product [20] then the only change is that we replace Witten's product by a modified product which just adds in the Moyal phases, and of course we replace $\alpha' \to \alpha'_{\rm eff}$.

Since the effective string scale $\alpha'_{\rm eff}$ is the same as that of non-commutativity θ, the noncommutative phases are non negligible only for energies of the order of those of string oscillators.

3.2 The free spectrum

In this subsection we will argue that the NCOS limit defines an open string theory on the 3-brane, as open string oscillators do not decouple in this limit. We will examine the spectrum in the free NCOS theory and see that the effective scale is indeed set by $\alpha'_{\rm eff}$.

We first consider the scaling limit in the NCYM picture. Near the NCYM limit one has weakly coupled closed strings coupled gravitationally to open strings. Open string excitations with string frame energies obeying $|g^{00}k_0^2| \ll 1/\alpha'$, or equivalently Einstein frame energies obeying $|g_E^{00}k_0^2| \ll m_p^2$, decouple from the closed strings. As $g^{00} = -1$, open string modes with $k_0 \ll 1/\sqrt{\alpha'}$ decouple from closed string modes. The decoupled theory includes all brane excitations with energies that obey this inequality, namely just the $\mathcal{N} = 4$ YM multiplet.

Now consider the same limit in the NCOS picture. The argument above ensures that open string modes with $k_0 \ll 1/\sqrt{\alpha'}$ decouple from closed strings. However, the open string oscillator states in this picture obey the mass shell condition set by the open string metric in the RHS of table 1

$$\frac{\alpha'_{\rm eff}}{\alpha'} k_A G^{AB} k_B = \frac{N}{\alpha'}\,, \tag{3.2}$$

with $A, B = 0, 1, 2, 3$. This implies

$$k^2 = \frac{N}{\alpha'_{\text{eff}}} \ll \frac{1}{\alpha'}\,, \tag{3.3}$$

with

$$\alpha'_{\text{eff}} = \frac{\theta}{2\pi} \tag{3.4}$$

in the limit $\alpha' \to 0$. Thus the decoupled theory on the brane includes all open string oscillator states! The mass spectrum is exactly the usual free spectrum on the three brane, except with α'_{eff} replacing α'.

3.3 Worldsheet correlators

Nontrivial vertex operators are functions of tangential worldsheet derivatives of X^A and normal worldsheet derivatives of X^M. Correlation functions of such vertex operators may be computed given the two point correlators of the free fields X^A restricted to the boundary of the world sheet, as well as the two point functions of the free fields X^M.

The boundary correlators of X^A are finite in the limit $\alpha' \to 0$, and are given by

$$X^A(0)X^B(\tau) = -\alpha'_{\text{eff}} G^{AB} \ln(\tau)^2 + \frac{i}{2}\theta^{AB}\epsilon(\tau)\,, \qquad A, B = 0, 1, 2, 3\,. \tag{3.5}$$

On the other hand, correlation functions involving the transverse directions X^M are derived from the sigma model

$$S = \frac{1}{4\pi\alpha'}\int G_{MN}\partial X^M \partial X^N = \frac{\alpha'_{\text{eff}} G_0^4}{4\pi\alpha'^2}\int \partial X^M \partial X^N \delta_{MN}\,. \tag{3.6}$$

In terms of the rescaled fields $Y^M = \frac{G_0^2 \alpha'_{\text{eff}}}{\alpha'} X^M$

$$S = \frac{1}{4\pi\alpha'_{\text{eff}}}\int \partial Y^M \partial Y^N \delta_{MN}\,. \tag{3.7}$$

The vertex operators representing physically normalized states are functions of the normal derivatives of Y^M.

Thus all correlation functions of NCOS vertex operators on the disk will be the same as in usual open string theory except that $\alpha' \to \alpha'_{\text{eff}}$ and we have extra non-commutative phases appearing as in [3]. The open string coupling constant is G_0 and it is finite.

4. The one loop diagram

One loop open string graphs usually contain closed string poles, and unitarity then requires that closed strings be included as asymptotic states. In this section we consider the nonplanar annulus diagram in the NCOS limit, and show that it has no physical closed string poles. This demonstrates that an on shell closed string cannot be produced in collisions of open strings.

Nonplanar diagrams for spatial Θ were computed in [11]–[16] — we will follow [14]. The nonplanar diagram for our case can be obtained by analytic continuation. For simplicity consider the case of two initial and two final open string tachyon vertex operators $V_T = G_o e^{ik_A X^A}$ in the bosonic string with incoming momenta k_1, k_2 and outgoing momenta k_3, k_4. Then we get for a D-3 brane in bosonic string theory [14, eq. 2.17].

$$
\begin{aligned}
\langle V_T(k_1)V_T(k_2)V_T(k_3)V_T(k_4)\rangle_{\text{annulus}} \sim & \\
\sim i\sqrt{G}G_o^4(4\alpha'_{\text{eff}})^{-2}\delta^4(k_1+k_2+k_3+k_4) & \\
\times \int_0^\infty \frac{ds}{2\pi s^{11}}\eta\left(\frac{is}{\pi}\right)^{-24} e^{-\frac{\alpha' s}{2}k_A g^{AB}k_B} \times & \\
\times \int_0^1 d\nu_1 d\nu_2 d\nu_3 d\nu_4 \Psi_1\Psi_2\Psi_{12} e^{\frac{i}{2}\left[k_3\times k_4(2\nu_{34}-\epsilon(\nu_{34}))-k_1\times k_2(2\nu_{12}-\epsilon(\nu_{12}))\right]}\,, & \qquad (4.1)
\end{aligned}
$$

with

$$
\begin{aligned}
&\Psi_1 = \left|\frac{\theta_{11}(\nu_{12}, is/\pi)}{\theta'_{11}(0,is/\pi)}\right|^{2\alpha'_{\text{eff}}k_1\cdot k_2}\,, \qquad \Psi_2 = \left|\frac{\theta_{11}(\nu_{34}, is/\pi)}{\theta'_{11}(0,is/\pi)}\right|^{2\alpha'_{\text{eff}}k_3\cdot k_4}\,,\\
&\Psi_{12} = e^{-s/4}\prod_{\substack{r=1,2\\ s=3,4}}\left|\frac{\theta_{10}(\nu_{rs}, is/\pi)}{\theta'_{11}(0,is/\pi)}\right|^{2\alpha'_{\text{eff}}k_r\cdot k_s}\,, \qquad (\nu_{rs}=\nu_r-\nu_s)\,,\\
&k=k_1+k_2\,, \qquad k_r\times k_s = k_{rA}\Theta^{AB}k_{sB}\,, \qquad k_r\cdot k_s = k_{rA}G^{AB}k_{sB}\,. \qquad (4.2)
\end{aligned}
$$

The expression for the annulus amplitude in (4.1) is written in the closed string channel. (The expression for the superstring would be similar except that the factor of $s^{-11} = s^{-d_t/2} \to s^{-3}$ in (4.1). This factor comes from the number of transverse dimensions d_t.) Closed string singularities arise in the integral over s in (4.1) as $\eta(is/\pi)$ may be expanded in a series in e^{-Ns}. We thus find non analyticities[4] (singularities) in the amplitude when

$$
\frac{\alpha'}{2}k_A g^{AB}k_B = -N\,. \qquad (4.3)
$$

In the NCOS scaling limit, this condition may be written as

$$
\frac{\pi\alpha'^2}{\theta G_o^4}k_\mu\eta^{\mu\nu}k_\nu + \frac{\theta}{4\pi}k_i\delta^{ij}k_j = -N\,. \qquad (4.4)
$$

Singularities on the real axis occur at a squared energy

$$
k_0^2 = k_1^2 + \left(\frac{G_0^2\theta}{2\pi\alpha'}\right)^2\left(k_2^2+k_3^2+\frac{2N}{\alpha'_{\text{eff}}}\right)
$$

[4]These singularites are 10 dimensional poles integrated over d_t transverse momenta.

that becomes arbitrarily large as α' is made increasingly small. In the strict limit $\alpha'{\to}0$, open string one loop amplitudes factorize on singularities of the form

$$\int d^{d_t}k_M \frac{1}{k_2^2+k_3^2+2N/\alpha'_{\rm eff}+g^{MN}k_Mk_N}\,.$$

As these singularities are never in the physical region, they do not correspond to physical states.[5] Recalling that G^{AB} is fixed in the NCOS limit, it is easy to see that the amplitude (4.1) is finite (except of course for the tachyon pole which is absent in the superstring). It is also straightforward using the results of [14] to show that there are no physical poles for any numbers of initial and final open string tachyon vertex operators. Higher mass vertex operators involve additional powers of the Green functions on the annulus. These are finite in the NCOS limit and so will not spoil the finiteness of the amplitudes. Although we have not worked out the details, we expect that the behavior of the superstring is similar.

It is instructive to contrast the behaviour of (4.1) in both the NCOS and the NCYM limits. In the latter case the $\alpha'\to 0$ limit is manifestly smooth when s/α' is held fixed. This forces one into a corner of the moduli space in which the massive open string states are decoupled [11]–[16]. In the NCOS limit (4.1) receives contributions from finite s, and so from all open string oscillator states. Apart from the non-commutative phases the one loop open string diagram (4.1) has almost the same form as the corresponding diagram in a theory with $B=0$, with α' replaced by $\alpha'_{\rm eff}$. However, the exponential term in (4.1) coming from momentum flowing along the closed string channel has a different α' dependence from standard string theory with zero B. This different dependence is responsible for the absence of physical closed string poles.

The absence of closed string poles in a non-commutative open string theory, whose non-commutativity parameter θ is $2\pi\alpha'_{\rm eff}$ as in our NCOS theories, may be understood more directly, as we explain below. This line of reasoning also suggests that a non commutative open string theory with $\theta<2\pi\alpha'_{\rm eff}$ has closed string poles, while the theory with $\theta>2\pi\alpha'_{\rm eff}$ is unstable.

Consider the simple non-planar diagram represented in figure 1, in an open string field theory. Let the open string theory in question be noncommutative, with noncommutativity parameter θ. The momentum integral for this diagram takes the form

$$\int d^4q e^{2ip\times q}I_{\theta=0}(q,p)\sim\int d^4q\int_0^\infty dte^{2ip\times q}e^{-2\pi\alpha'_{\rm eff}tq^2+t\beta p\cdot q+\cdots} \qquad (4.5)$$

[5] These singularities $\sim (k_i^2)^{\frac{d_t-2}{2}}\ln(k_i^2)$ are very similar to those induced by one loop graphs in spatially noncommutative field theories, as found in [21, 22]. Notice that if $d_t\geq 2$ (p branes with $p<7$ in the supersymmetric case), this amplitude, though non analytic, is finite at $k=0$. For $d_t\leq 2$ the amplitude diverges at $k_i^2=0$. It is possible that stronger IR singularities appear at higher loops, specially for high dimensional branes.

where $I_{\theta=0}(p,q)$ is the integrand at $\theta = 0$ and $p \times q = p_\mu q_\mu \Theta^{\mu\nu}/2$. We have exponentiated the propagators in the diagram using a Schwinger proper time representation, where t is the total proper time along the loop and we have explicitly given the form of the leading dependence on q (β is some other Schwinger parameter, which is also integrated over; we have supressed this integral in (4.5) for simiplicity). When q is integrated over we get the diagram as a function of t and β. As in [21], the effect of noncommutativity on this integral is an extra term in the exponent of the form

$$e^{-p \circ p/(8\pi\alpha'_{\text{eff}} t)}\,. \tag{4.6}$$

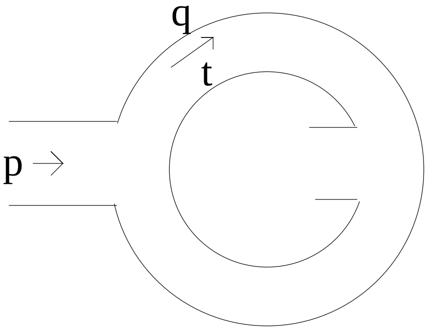

Figure 1: Nonplanar open string diagram. In open string field theory we would build it from the cubic vertex and we would consider states carrying momentum q and $q+p$ along the loop.

where $p \circ p = -p_\mu \Theta^{\mu\nu}\Theta_{\nu\rho}p^\rho = -\theta^2 p^2$. This may be seen by shifting the integral over q to one over $q'_\mu = q_\mu + i\Theta_{\mu\nu}p^\nu/(4\pi\alpha'_{\text{eff}} t)$. Note that terms of the form $q \cdot p$ in (4.5) are unaffected by the shift due to the antisymmetry of Θ.

Thus the integrand of (4.5) is modified from its $\theta = 0$ value only by the additional exponential factor (4.6). On shifting to the $s = \pi/t$ channel, the integrand has the usual terms of the form $e^{-s\frac{\alpha'_{\text{eff}}}{2}(-p_0^2+p_1^2+\cdots)}$ (terms that would produce the s-channel poles if θ were zero) multiplied by the additional factor $e^{-s\frac{\theta^2}{8\pi^2\alpha'_{\text{eff}}}(p_0^2-p_1^2)}$. When $\theta = 2\pi\alpha'_{\text{eff}}$ this extra factor exactly cancels the p_0, p_1 dependence of the exponent. Here we have used the fact that we are in lorentzian signature so that the final sign of the exponent in (4.6) is the opposite to the one in euclidean signature. If θ is slightly less that its critical value, then (4.6) does not cancel the closed string poles. If θ is bigger than its critical value then all closed string poles turn tachyonic, a reflection of the instability of the system.

5. Higher loop diagrams

In this section we will examine higher loop string diagrams in the NCOS limit. We will not attempt to prove that the limit is nonsingular for arbitrary diagrams, but we will observe that a simple counting of powers of α' does not reveal any difficulties. Naively, a genus g surface in the string loop expansion is weighted by g_{str}^{2g-2}. As g_{str} diverges in the NCOS limit, a perturbative expansion in genus seems impossible. However, we shall argue below that both holes and handles are really weighted by powers of G_0 and so high genus surfaces are suppressed at weak open string coupling.

5.1 Holes

The addition of a hole in the world sheet is accompanied by one power of $g_{\rm str}$. It also leads to an additional integral over the zero mode momentum circulating around the loop. As shown in [6], these integrals have a measure factor proportional to $\det^{1/2}(g + 2\pi\alpha' B)\det^{-1/2}(g)$. Hence the total weighting of a hole is

$$g_{\rm str}\frac{\det^{1/2}(g + 2\pi\alpha' B)}{\det^{1/2}(g)} = G_o^2\,, \tag{5.1}$$

and is finite as $\alpha' \to 0$.

5.2 Handles

Consider an open string world sheet A, with open string boundary conditions corresponding to a 3-brane. The amplitude on a worldsheet (B) with an additional handle can be factorized in the closed string channel along the handle. The resultant amplitude reads schematically as

$$S_B = \sum S_{A_{V_a,V_a}}\lambda_{\rm eff}^2 \int d^6k \frac{1}{g^{IJ}k_I k_J + m_a^2}\,, \qquad (I,J = 0,\ldots,9)\,.$$

Here $S_{A_{V_a,V_a}}$ denotes the amplitude on A with two extra closed string insertions. The integral is over the momenta of the intermediate states in the transverse directions (momentum is not conserved in these directions). The effective coupling $\lambda_{\rm eff}^2$ is determined as follows: A closed string mode ϕ with spacetime action

$$S = \frac{1}{g_{\rm str}^2\alpha'^4}\int d^{10}x\sqrt{g}(\partial_I\phi\partial_J\phi g^{IJ} + m_a^2\phi^2)$$

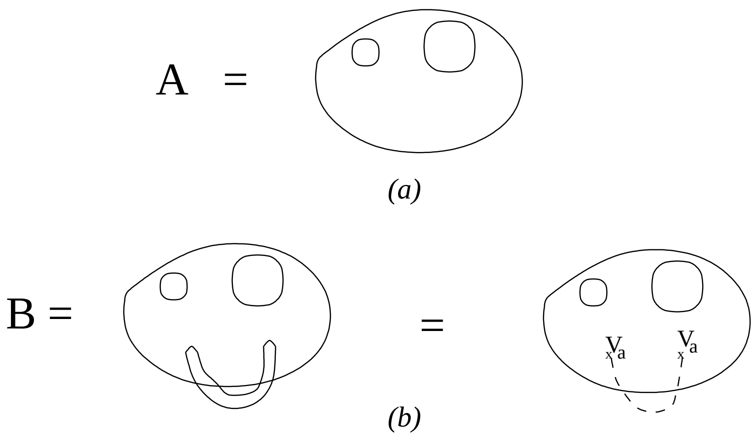

Figure 2: Adding a handle to a worldsheet A, we obtain a worldsheet B, which can be represented as coming from the propagation of closed string states between two points of the worldsheet. We sum over all closed string states.

has effective coupling

$$\lambda_{\rm eff} = \frac{g_{\rm str}\alpha'^2}{g^{1/4}} = \frac{\alpha'^{5/2}}{G_0^4 \alpha'^{1/2}_{\rm eff}}$$

in the NCOS limit. The integral

$$\int d^6k \frac{1}{g^{IJ}k_I k_J + m_a^2} = \alpha' \int d^6k \frac{1}{\frac{\alpha'^2 k_M k_N \delta^{MN}}{\alpha'_{\rm eff} G_0^4} + N + \alpha'_{\rm eff}(k_2^2 + k_3^2) + \cdots}$$

is of order

$$\alpha' \left(\frac{\alpha'_{\rm eff} G_0^4}{\alpha'^2} \right)^3 .$$

Finally, in the normalization we have adopted, $S_{A_{V_a,V_a}}$ is of the same order as S_A.

Putting it all together, we find that

$$\frac{S_B}{S_A} \approx G_0^4 \alpha'^2_{\rm eff} . \tag{5.2}$$

Thus we conclude that extra handles, in the NCOS limit, are neither infinitely suppressed nor enhanced in the NCOS limit. They are instead really weighted by a factor of G_0^4, as they would have been for an ordinary weakly coupled open string theory.[6]

6. Supergravity duals

The considerations of the previous sections generalize to open string theories on N coincident 3-branes. In that case since we are dealing with a deformation of $\mathrm{U}(N)$ $\mathcal{N} = 4$ SYM we expect that it should have a supergravity dual for large N. The relevant supergravity solutions were written in [24, 25]. We start from the lorentzian version of the solution [25, eq. (2.3)], with $B_{23} = 0$. Then we do the following scaling of parameters

$$\begin{aligned}
r &= \sqrt{\alpha'} u \\
\cosh\theta' &= \frac{\widetilde{b}'}{\alpha'} \\
g &= \frac{\widetilde{g}\widetilde{b}'}{\alpha'} \\
x_{0,1} &= \frac{\widetilde{b}'}{\sqrt{\alpha'}} \widetilde{x}_{0,1} \\
x_{2,3} &= \sqrt{\alpha'} \widetilde{x}_{2,3} \\
R^4 &= \text{fixed} = 4\pi \widetilde{g} N
\end{aligned} \tag{6.1}$$

[6]See also [23] for a discussion of diagrams with many holes.

We obtain[7]

$$
\begin{aligned}
ds^2_{\rm str} &= \alpha' f^{1/2}\left[\frac{u^4}{R^4}(-d\widetilde{x}_0^2 + d\widetilde{x}_1^2) + f^{-1}(d\widetilde{x}_2^2 + d\widetilde{x}_3^2) + du^2 + u^2 d\Omega_5^2\right] \\
2\pi\alpha' B_{01} &= \alpha' \frac{u^4}{R^4}\,, \\
e^{2\phi} &= \widetilde{g}^2 f \frac{u^4}{R^4} \\
A_{23} &= \alpha' \frac{1}{\widetilde{g}} f^{-1}\,, \\
F_{0123u} &= \alpha'^2 \frac{1}{\widetilde{g}} \frac{4f^{-1}}{u} \\
f &= 1 + \frac{R^4}{u^4}\,.
\end{aligned} \tag{6.2}
$$

The particular scalings that we have to do to reproduce this solution are, up to constants, the same as those in section 2.2. The only scaling that is not so obvious is the scaling of the radial coordinate. Notice that in the $\mathcal{N}=4$ SYM case we rescale the radial coordinate as $r \sim \alpha' u$. The fact that we have $r \sim \sqrt{\alpha'}u$ in this case is related to the fact that the closed string metric has a factor of $1/\alpha'$ in section 2.2. We see from (5.2) that for small u we recover the usual $AdS_5 \times S^5$ solution as we expect, since the open string theory reduces to $\mathcal{N}=4$ SYM at low energies. In particular we see that we should identify $\widetilde{g} = G_0^2$. As we increase u the metric becomes different than the metric of AdS and we also see that the dilaton becomes large. This suggests that for large u we should do an S-duality to analyze the solution. After we do the S-duality we obtain a solution which is the same as the supergravity solution which corresponds to a D3 brane with spatial non-commutativity in the directions 23, see [25, eq. (2.7)]. This suggests that at very high energies the open string theory we are studying would have a dual description in terms of the theory with spatial non-commutativity.

7. Discussion

7.1 Open string dipoles and UV/IR

Free open string states in the NCOS limit behave quite differently from ordinary open strings propagating in the same metric, despite having the same spectrum. In the presence of background fields, (as discussed for example in [7, 8, 9] and especially in [14] for the magnetic case) the mode expansion reads

$$X^\mu(\sigma,\tau) = x_0^\mu + 2i\alpha'_{\rm eff} p^\mu \tau + \frac{1}{\pi}\Theta^{\mu\nu} p_\nu \sigma + (\text{oscillators})\,. \tag{7.1}$$

[7]Here we normalize the B field as in the previous sections, in [25] it was normalized differently by a factor of $2\pi\alpha'$, $B_{MR} = 2\pi\alpha' B_{here}$.

For strings in the NCOS limit this implies that the distance along the direction of the field between the two ends of the string, as measured in the metric G^{AB}, is

$$\Delta X^1 = 2\pi\alpha'_{\rm eff} k_0\,, \tag{7.2}$$

plus oscillator contributions which time average to zero. (Note that ΔX^1 is the distance between the endpoints of the string worldsheet along a line of constant worldsheet time rather than along a line of constant X^0. As we argue below, the proper length of the string is given by a formula analogeous to (7.2) with k_0 replaced by the centre of mass energy of the state.)

The invariant energy and proper length of an oscillator state may be estimated as follows. The tension of an open string aligned with a near critical electric field is almost canceled by a negative contribution from its dipole interaction with the field. In the NCOS limit, the effective tension $T_{\rm eff} = 1/4\pi\alpha'_{\rm eff}$ (see (2.11)). The energy of such a string, with an oscillation number N, is $E = T_{\rm eff}L + \pi N/L$. This is minimized for $L = 2\pi\sqrt{\alpha'_{\rm eff}N} = 2\pi\alpha'_{\rm eff}E$, as in (7.2).

7.2 Thermodynamics

At low energies, compared to $1/\sqrt{\alpha'_{\rm eff}}$ the NCOS theory reduces to ordinary $\mathcal{N}=4$ SYM, and its free energy scales like T^4. At intermediate energies, the thermodynamics of a weakly coupled NCOS theory ($\lambda = G_o^2 N \ll 1$), may be expected to reflect its Hagedorn density of states.

However, as argued in this paper, the weakly coupled NCOS theory has a dual description as a strongly coupled NCYM theory. In a spatially noncommutative field theory, at weak coupling, planar diagrams [26] dominate over nonplanar diagrams [21] for energies $k_0 \gg 1/\sqrt{\theta}$. It is plausible that this result to continues to hold at strong coupling,[8] with a crossover scale renormalized by a function of the coupling. If true, this assertion implies that, at high temperatures, the free energy of spatially noncommutative SYM is proportional to the free energy of ordinary large-N SYM, and so scales with temperature like T^4, even at large $G_o'^2$.

It would be interesting to investigate this issue further.

7.3 Generalizations to other dimensions

In this paper we have 'derived' the existence of a decoupled four dimensional open string theory, NCOS, by S dualizing spatially noncommutative SYM. We presented evidence that, independent of this derivation, the resultant theory is well defined, and weakly coupled over a range of parameters.

It is easy to extend our construction of the NCOS to other dimensions, even though we do not have an independent (S duality) argument for the decoupling of

[8]This statement is true at least in the 'supergravity' limit $\lambda \gg 1$, $G_o^2 \ll 1$; in that limit [25, 24], supergravity suggests that planar diagrams dominate for $k_0 \gg 1/(\lambda\theta^2)^{1/4}$.

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closed strings. The NCOS scaling limit for a p brane is, once again, defined by table 1, where the indices i, j run from $2, \ldots, p$ and A, B from $0, \ldots, p$. In other words, this limit still describes a near critical electric field turned on in the 1 direction. The open string coupling defined in (2.9) and the effective low energy Yang-Mills coupling constant $g_{\mathrm{YM}}^2 \sim G_0^2 \alpha'^{(p-3)/2}_{\mathrm{eff}}$ are finite. In the NCOS limit, open strings appear to decouple from closed strings for all p. The annulus amplitude is finite in arbitrary dimension, and always factorizes on unphysical closed string poles. As in the 3-brane, string diagrams with handles and holes are suppressed by powers of the open string coupling, and may be neglected at weak coupling.

In fact, these open string theories appear to be non-gravitational UV finite completions of low energy (supersymmetric) Yang-Mills. This statement appears to be true even in high dimensions where the gauge theory is non-renormalizable.

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